p.450 # 1-8, 9-19 odds

p450

~~13~~, ~~19~~, 17, ~~9~~, 15,
~~11~~, ~~6~~

2) 5

4) 1/3

6) 1/4

8) 1/6

11.) a. $\lim_{x\to 0^-} \frac{\sin x}{x^3} \Rightarrow \left(\frac{0}{0}\right)$

$\lim_{x\to 0^-} \frac{\cos x}{3x^2} \Rightarrow \left(\frac{1}{0}\right)$

∞

b. Same thing

∞

9.) a) $\lim_{x\to 0^-} \frac{\sin 4x}{\sin 2x} = \frac{0}{0}$

$\lim_{x\to 0^-} \frac{4\cos 4x}{2\cos 2x} = \frac{4}{2} = 2$

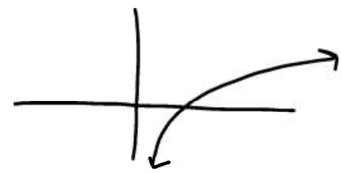
b) $\lim_{x\to 0^+} \frac{4\cos 4x}{2\cos 2x} = \frac{4}{2}$ 2 2(c)

15 $\displaystyle \lim_{x\to\infty} \frac{\ln(x+1)}{\log_2 x} = \frac{\infty}{\infty}$

$$\lim_{x\to\infty} \frac{\frac{1}{x+1}}{\frac{1}{x\ln 2}} = \lim_{x\to\infty} \frac{x\ln 2}{x+1} = \lim_{x\to\infty} \frac{\ln 2}{1}$$

$= \ln 2$

17 $\displaystyle \lim_{x\to 0^+} (x \ln x) = 0 \cdot -\infty$



$$\lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \frac{-\infty}{\infty}$$

$$\lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x\to 0^+} (-x) = 0$$

19 $\displaystyle \lim_{x\to 0^+} (\overset{\infty}{\csc x} \overset{-}{-} \overset{\infty}{\cot x} \overset{+}{+} \overset{1}{\cos x})$

$$\lim_{x\to 0^+} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} + \frac{\cos x \cdot \sin x}{\sin x}\right)$$

$$\lim_{x\to 0^+} \left(\frac{\overset{1}{1} \overset{-}{-} \overset{1}{\cos x} \overset{+}{+} \overset{1}{\cos x} \overset{\cdot}{\cdot} \overset{0}{\sin x}}{\sin x}\right) \quad \frac{0}{0}$$

$$\lim_{x\to 0^+} \frac{\sin x + \cos x(\cos x) + \sin x(-\sin x)}{\cos x}$$

$$\lim_{x\to 0^+} \frac{\overset{0}{\sin x} + \overset{1}{\cos^2 x} - \overset{0}{\sin^2 x}}{\underset{1}{\cos x}} = \frac{1}{1} = 1$$

More 8.2: Indeterminate forms: 1^∞, 0^0, ∞^0

$$e^{\ln b} = b \quad , \quad e^{\ln f(x)} = f(x)$$

If $\lim_{x\to a} \ln f(x) = L$, then

$\lim_{x\to a} e^{\ln f(x)} = e^L$, then

$\lim_{x\to a} f(x) = e^L$

① $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x$ $\quad \left(1+\frac{1}{\infty}\right)^\infty = (1+0)^\infty = 1^\infty$

First: find $\lim_{x\to\infty} \ln\left(1+\frac{1}{x}\right)^x$

$\lim_{x\to\infty} x \cdot \ln\left(1+\cancel{\frac{1}{x}}^{\,0}\right)$ $\quad \infty \cdot 0$

$\lim_{x\to\infty} \dfrac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}$ $\quad \frac{0}{0}$

$$\lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x}} \cdot \frac{-1}{x^2}}{\frac{-1}{x^2}} = \lim_{x\to\infty} \frac{1}{1+\cancel{\frac{1}{x}}^{\,0}} = 1$$

Because $\lim_{x\to\infty} \ln\left(1+\frac{1}{x}\right)^x = 1$

$\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = e^1 = e$

② Find $\lim_{x\to 0^+} x^x$

First, find $\lim_{x\to 0^+} \ln x^x = \lim_{x\to 0^+} x \cdot \ln x$ $\quad 0 \cdot -\infty$

$= \lim_{x\to 0^+} \dfrac{\ln x}{\frac{1}{x}}$ $\quad \frac{-\infty}{\infty}$ $\quad = \lim_{x\to 0^+} \dfrac{\frac{1}{x}}{-\frac{1}{x^2}} =$

$= \lim_{x\to 0^+} -x = 0$

$\therefore \lim_{x\to 0^+} x^x = e^0 = 1$

HW: p450 # 21-37 odds, 43, 45